TIFR/TH/15-09

Fermion localization and flavour hierarchy in higher curvature spacetime

Sayantan Choudhury *
Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai - 400005, India †

Joydip Mitra ‡
Department of Physics, Scottish Church College, 1 & 3 Urquhart Square, Kolkata - 700 006, India

Soumitra SenGupta §
Department of Theoretical Physics, Indian Association for the Cultivation of Science, 2A & 2B Raja S. C. Mullick Road, Kolkata - 700 032, India.

Fermion localization in a braneworld model in presence of dilaton coupled higher curvature Gauss-Bonnet bulk gravity is discussed. It is shown that the left handed fermionic modes can be localized on the visible brane due to the dilaton coupled higher curvature term without the necessity of any external localizing bulk field. This offers a natural resolution of the flavour hierarchy problem in Standard Model.

Randall Sundrum warped geometry model [1] is an eminently successul model in resolving the long standing gauge hierarchy/naturalness problem in an otherwise successful Standard model of elementary particles. This resulted in extensive search for a signature of RS model in LHC [2–7] . When applied as a physics beyond Standard Model (SM) such a scenario is often based on an underlying assumption that the SM fermions in general can propagate in the bulk while their chiral states are appropriately localized in different regions of bulk spacetime producing the desired 4-dimensional fermion masses on the visible brane. Though RS model itself can not provide any justification for this localization, there have been efforts to provide an explanation of this by introducing an ad-hoc scalar field in the anti-de Sitter bulk of RS model [8–14]. Such a mechanism of localization however gives rise to the speculation about the possible back-reaction of the scalar to jeopardize the original RS solution for the warp factor. The origin of hierarchy of fermion masses in Standard model is a problem yet to be resolved. In this work we show that a string inspired modification to Einstein gravity via dilaton and higher curvature effects can explain the hierarchy among the fermion masses, where the effective masses of fermions are determined by their couplings with the dilaton field. Our work is staged on a five dimensional spacetime where our universe sits at one of the fixed points of the orbifolded extra spacetime dimension. This work thus is an attempt to look for an alternative pathway for this localization which resorts to the presence of higher curvature corrections to the classical Einsteinian gravity in the bulk as postulated in RS model. Such a correction, though heavily suppressed in low energy world, assumes significance in an AdS bulk with curvature $\sim$ Planck scale as assumed in RS model. At the leading order, such correction over Einstein Gravity which is free from the appearance of any ghost field due to the higher derivative terms, is the Gauss-Bonnet gravity where the various quadratic curvature terms appear in suitable combination to make the theory stable. Such a correction is also inspired by string theory which in addition also predicts the presence of the scalar dilaton in the action.

Following the path adopted in RS model, if the dilaton coupled GB gravity is compactified on S^1/Z_2 orbifold, it gives rise to two branches of warped solutions, one of which is stable and free of ghost. In this work we adopt this ghost free branch and study the effects of higher curvature couplings on the two chirality states of 5 dimensional SM fermions to study their localization on the standard model 3-brane.

As discussed earlier, if the standard model fermions also propagate in the bulk, just as graviton, then by appropri-

* Electronic address: sayantan@theory.tifr.res.in, sayanphysicsisi@gmail.com
† Presently working as a Visiting (Post-Doctoral) fellow at DTP, TIFR, Mumbai.
‡ Electronic address: jmphys@scottishchurch.ac.in
§ Electronic address: tpssg@iacs.res.in

ately choosing the interaction potential between the fermion and a localizing field one can try to obtain the appropriate overlap between the chiral states of the fermionic wave functions to localize the left chiral massless fermion states on the brane. However without resorting to any such external ad-hoc field can one produce this desired feature through the higher curvature GB terms in the bulk? This would then provide a natural explanation of observing only the left-handed neutrinos in our universe while the massive fermions appear with both the chiral states. We try to address this question in the present work in the framework of GB-dilaton induced warped geometry model [15].

In this work, we consider a bulk fermion (massless and massive) and study the localization profile in the five dimensional warped geometry model in the backdrop of Gauss-Bonnet dilaton gravity. We will show that inclusion of higher curvature terms lead to the localization of left handed fermionic modes near the visible brane as their mass decreases while the right hand modes are localized within the bulk. In such localization scenario therefore one does not need to invoke an external bulk field as has been proposed earlier. Since the Standard model only includes the left handed modes of massless fermions, we can clearly see that this higher curvature setup will automatically allow the fermionic wave functions to localize themselves in the TeV brane. On the other hand, the delocalization of right handed modes inside the bulk can produce interesting phenomenological consequences like fermion mass generation as suggested by Ref. [14].

Before going to discuss the various features of fermion localization, we start with the 5D action for the two brane warped geometry model including higher curvature gravity as [15]:

$$
\begin{aligned}
S = \int d^5x \Bigg[\sqrt{-g_{(5)}} \Bigg\{ \frac{M^3_{(5)}}{2} R_{(5)} + \frac{\alpha_{(5)} M_{(5)}}{2} \left[R^{ABCD(5)} R^{(5)}_{ABCD} - 4 R^{AB(5)} R^{(5)}_{AB} + R^2_{(5)} \right] \\
+ \frac{g^{AB}}{2} \partial_A \chi(y) \partial_B \chi(y) - 2\Lambda_5 e^{\chi(y)} \Bigg\} - \sum_{i=1}^{2} \sqrt{-g^{(i)}_{(5)}} T_i \delta(y - y_i) \Bigg]
\end{aligned}
\tag{1}
$$

where $A, B, C, D = 0, 1, 2, 3, 4$. Here i signifies the brane index, $i = 1$(hidden), 2(visible) and T_i is the brane tension. Additionally α_5 and $\chi(y)$ represent the Gauss-Bonnet coupling and dilaton. The background metric describing slice of the $\mathbf{AdS_5}$ is given by,

$$
ds^2_5 = g_{AB} dx^A dx^B = e^{-2k_{\mathbf{M}}(y) r_c |y|} \eta_{\alpha\beta} dx^\alpha dx^\beta + r_c^2 dy^2 \tag{2}
$$

where r_c represents the compactification radius of extra dimension. Here the orbifold points are $y_i = [0, \pi]$ and in the above metric ansatz $\eta_{\alpha\beta} = diag(-1, +1, +1, +1)$ is flat Minkowski metric. The explicit form of warp factor is given by [15]:

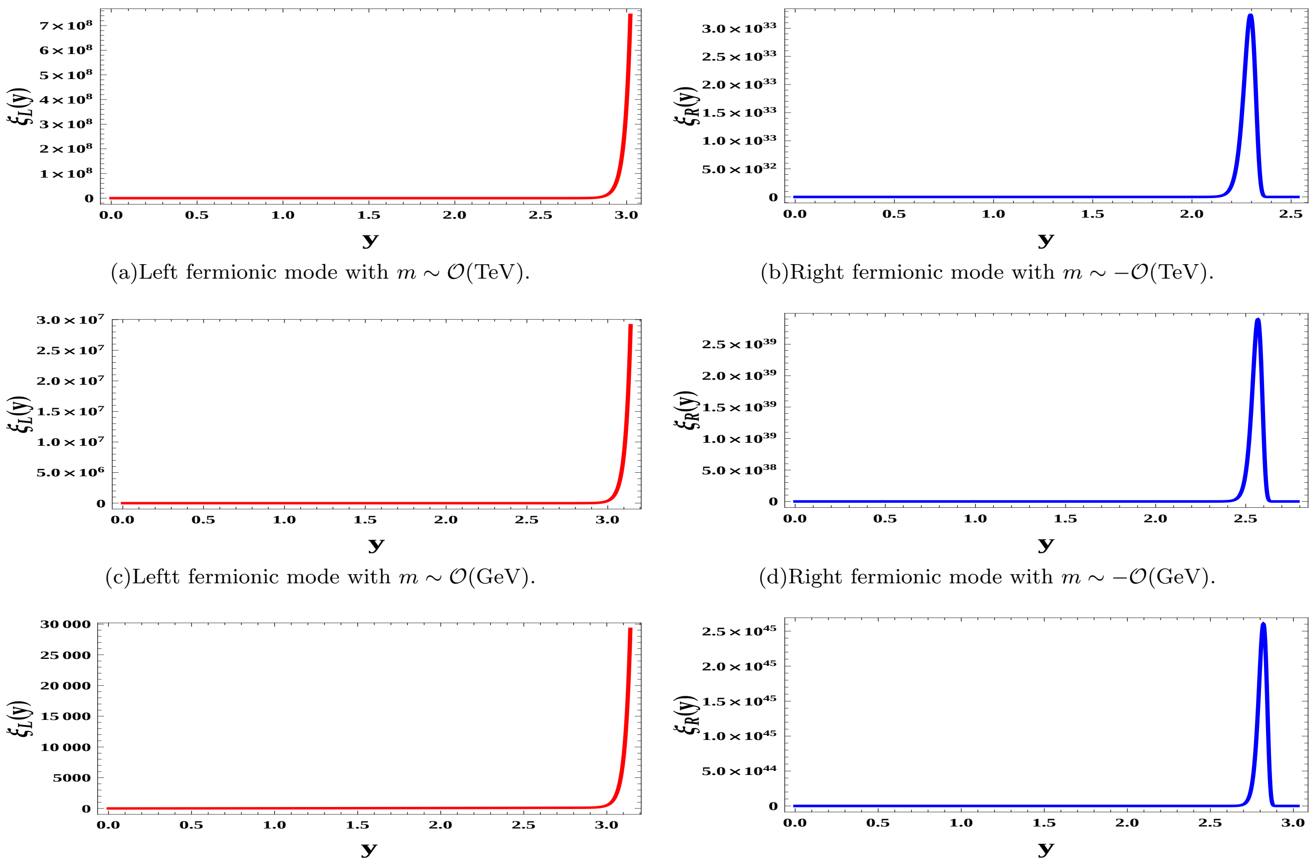
$$
k_{\mathbf{M}}(y) = k_{RS}\ e^{\frac{\chi(y)}{2}} \left[1 + \mathbf{L} + \mathcal{O}(\mathbf{L}^2) \right]. \tag{3}
$$

where $\mathbf{L} := \frac{4\alpha_{(5)} k^2_{RS}}{M^2_{(5)}}$ in which $M_{(5)}$ signifies 5D mass scale and $k_{RS} = \sqrt{-\frac{\Lambda_5}{24 M^3_{(5)}}}$. Here we neglect the possibility of getting other branch of solution for the warp factor, which leads to the spurious ghost degrees of freedom. Additionally, the dilaton satisfies the linear profile in the bulk, $\chi(y) = c_1|y|$, where c_1 is an arbitrary integration constant. As the nature of warping influences the localization profile of the bulk fermion, therefore it is expected that the dilaton charge c_1 for a given fermionic field will determine the localization property and hence the effective fermion mass term on the brane.

We will now start our discussion regarding the localization scenario of fermionic modes. The five dimensional action for the massive fermionic field can be written as:

$$
\begin{aligned}
S_f &= \int d^5x \left[Det(\mathcal{V}) \right] \ \left\{ i\bar{\Psi}(x,y) \gamma^\alpha \mathcal{V}^M_\alpha \overleftrightarrow{\mathbf{D}}_\mu \Psi(x,y) \delta^\mu_M - sgn(y) m_f \bar{\Psi}(x,y)\Psi(x,y) + \ h.c. \right\} \\
&= \int d^5x\ e^{-4k_{\mathbf{M}}(y) r_c |y|} \left\{ \bar{\Psi}(x,y) \left[i e^{k_{\mathbf{M}}(y) r_c |y|} \gamma^\mu \partial_\mu + \gamma^5 (\partial_y - 2r_c \partial_y \{k_{\mathbf{M}}(y)|y|\}) - sgn(y) m_B \right] \Psi(x,y) + h.c. \right\}
\end{aligned}
\tag{4}
$$

where $\overleftrightarrow{\mathbf{D}}_\mu := \left(\overleftrightarrow{\partial}_\mu + \Omega_\mu \right)$ represents the covariant derivative in presence of fermionic spin connection $\Omega_\mu = \frac{1}{8} \omega^{\hat{A}\hat{B}}_\mu \left[\Gamma_{\hat{A}}, \Gamma_{\hat{B}} \right]$. Here $\omega^{\hat{A}\hat{B}}_\mu$ represents the gauge field respecting $\mathcal{SO}(3,1)$ transformation on the vierbein coordinate. Here we assume that the bulk fermion mass m_B originates through an underlying spontaneous symmetry breaking in bulk via 5-dimensional Higgs mechanism [16, 17]. The 5D Gamma matrices $\Gamma^{\hat{A}} = \left(\gamma^\mu, \gamma_5 := \frac{i}{4!} \epsilon_{\mu\nu\alpha\beta} \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta = i\gamma_4 \right)$



(a)Left fermionic mode with $m \sim \mathcal{O}(\text{TeV})$.

(b)Right fermionic mode with $m \sim -\mathcal{O}(\text{TeV})$.

(c)Leftt fermionic mode with $m \sim \mathcal{O}(\text{GeV})$.

(d)Right fermionic mode with $m \sim -\mathcal{O}(\text{GeV})$.

(e)Left fermionic mode with $m \sim \mathcal{O}(\text{MeV})$.

(f)Right fermionic mode with $m \sim -\mathcal{O}(\text{MeV})$.

FIG. 1: Localization of the left and right handed fermionic profile $\xi_{L,R}(y)$. For all situations we have taken $\alpha_{(5)} = 10^{-3}$, $c_1 = 0.17$, $m_B = (+1(left), -1(right))$ which fixes, $k_{\mathbf{M}} r_c = 12$, is necessarily required to solve the hierarchy problem. Here all the masses are given in the Planckian unit.

satisfy the Clifford algebra anti-commutation relation $\{\Gamma^{\hat{A}}, \Gamma^{\hat{B}}\} = 2\eta^{\hat{A}\hat{B}}$ with $\eta^{\hat{A}\hat{B}} = diag\,(-1,+1,+1,+1,+1)$. In this context $g_{MN} := \left(\mathcal{V}^{\hat{A}}_{M} \otimes \mathcal{V}^{\hat{B}}_{N}\right)\eta_{\hat{A}\hat{B}}$, where $\mathcal{V}^{\hat{A}}_{M}$ are characterized by the usual conditions:

$$\mathcal{V}^{4}_{4} = 1, \quad \mathcal{V}^{\hat{A}}_{\mu} = e^{k_{\mathbf{M}}(y) r_c |y|}\delta^{\hat{A}}_{\mu}, \quad Det(\mathcal{V}) = e^{-4k_{\mathbf{M}}(y) r_c |y|} \tag{5}$$

and $\hat{A}, \hat{B}$ being tangent space indices. For our set up $\mathcal{SO}(3,1)$ spin connection can be written as:

$$\Omega_4 \;=\; 0, \quad \Omega_\mu = -\frac{1}{2} e^{-k_{\mathbf{M}}(y) r_c |y|} k_{\mathbf{M}}(y) r_c \gamma_5 \gamma_\mu. \tag{6}$$

We decompose the five-dimensional spinor as $\Psi(x,y) = \psi(x)\xi(y)$. In the massless case the definite chiral states $\psi_L(x)$ and $\psi_R(x)$ correspond to left and right chiral states in four dimension. The ψ_L and ψ_R are constructed by, $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma^5)\psi$. Here ξ denotes the extra dimensional component of the fermion wave function. We then can decompose five-dimensional spinor in the following way:

$$\Psi(x,y) = \psi_L(x)\xi_L(y) + \psi_R(x)\xi_R(y) \tag{7}$$

Substituting the above decomposition in Eq (4) we obtain the following equations for the fermions,

$$e^{-k_{\mathbf{M}}(y) r_c |y|}\left[\pm(\partial_y - 2r_c \partial_y\{k_{\mathbf{M}}(y)|y|\}) + sgn(y) m_B\right]\xi_{R,L}(y) = -m\;\xi_{L,R}(y) \tag{8}$$

where m_B and m represent the 5D bulk mass and 4D effective mass of fermion respectively. The 4D fermions obey the canonical equation of motion, $i\gamma^\mu \partial_\mu \psi_{L,R} = m\psi_{L,R}$. Also it is important note that the left and right handed part

4D mass m (in GeV)	5D bulk mass $m_{\mathbf{B}}$ (in M_{Pl})	GB coupling $\mathbf{L}$	Dilaton coupling c_1
10^3	1	$10^{-3}-10^{-7}$	0.007
1	1	$10^{-3}-10^{-7}$	0.033
10^{-3}	1	$10^{-3}-10^{-7}$	0.057
10^{-6}	1	$10^{-3}-10^{-7}$	0.078
10^{-9}	1	$10^{-3}-10^{-7}$	0.098

TABLE I: Parameter space required to generate overlap of left and right handed fermion wavefunctions at the visible brane via 4D effective mass.

of the extra dimensional wave function satisfies the usual ortho-normality condition. Finally, the solution of Eq (8) turns out to be:

$$\xi_{L,R}(y) = \mathcal{N}\exp\left[2\, e^{\frac{\chi(y)}{2}} k_{\mathbf{M}}(y)|y|r_c \pm m\int dy\; e^{k_{\mathbf{M}}(y)r_c|y|} \pm sgn(y)m_B|y|\right] \tag{9}$$

where $\mathcal{N}$ represents the normalization constant [1].

Fig. (1(a)-1(f)) describe the localization profiles of the fermion wave function inside the bulk. They clearly depict that for both massive as well as massless fermions, the left handed mode is localized on the brane while the right handed fermions are localized inside the bulk. Additionally, from the prescribed analysis we can observe that the gradual increment in the dilaton coupling c_1 for a fixed value of Gauss Bonnet parameter $\mathbf{L}$ within the window $10^{-3} < \mathbf{L} < 10^{-7}$ will shift the peak position of the right handed fermionic wavefunction towards left side of the visible brane towards the bulk. In such a situation the hight of the right fermionic mode increases, amount of localization increases and the localization position of the left fermionic mode slightly shift from the visible brane towards the bulk. We also observe that the the effective 4D mass m term decreases as the peak position of the left handed fermionic mode shifts towards visible brane. The overlap wavefunction of the left and right handed mode on the visible brane determines the effective mass of the fermion on the 3 brane. The effective 4D mass can be computed from the overlap integral as:

$$\mathbf{I}_{overlap} = m_B\int d^5x\,[Det(\mathcal{V})]\;\, sgn(y)\bar{\Psi}(x,y)\Psi(x,y) \;\;= \int d^4x\; m_{L,R}\left[\bar{\Psi}_L(x)\Psi_R(x) + \bar{\Psi}_R(x)\Psi_L(x)\right] \tag{10}$$

where the 4D effective mass $m_{L,R}$ is given by:

$$m_{L,R} = m_B\int_0^{\pi} dy\; e^{-4k_{\mathbf{M}}(y)r_c|y|} sgn(y)\xi_L^{\dagger}(y)\xi_R(y). \tag{11}$$

Further substituting Eq (9) in Eq (11) the effective mass $m = m_{L,R}$ can be recast as:

$$m = 2m_B\frac{\sqrt{5\left[1+\mathbf{L}+\mathcal{O}(\mathbf{L}^2)\right]}\exp\left[\left\{48\pi\; e^{\frac{c_1\pi}{2}}\left(e^{\frac{c_1\pi}{2}}-1\right)+\frac{6}{5c_1}\right\}\left[1+\mathbf{L}+\mathcal{O}(\mathbf{L}^2)\right]\right]}{\sqrt{\frac{\pi}{6c_1}}\left(\text{Erfi}\left[\frac{\sqrt{6}\sqrt{[1+\mathbf{L}+\mathcal{O}(\mathbf{L}^2)]}}{\sqrt{5c_1}}\right]-\text{Erfi}\left[\frac{\sqrt{6}\sqrt{[1+\mathbf{L}+\mathcal{O}(\mathbf{L}^2)](1+5c_1\pi)}}{\sqrt{5c_1}}\right]\right)} \tag{12}$$

---

[1] Applying the normalization of the extra dimensional wave function for left and right chiral fermionic modes the normalization constant can be expressed as:

$$\mathcal{N} = \frac{1}{\sqrt{\int_0^{\pi} dy\;\, \exp\left[\left(4e^{\frac{\chi(y)}{2}}-3\right)k_{\mathbf{M}}(y)r_c|y|\right]}}.$$

where we fix $k_{RS}r_c = 12$, which is necessary condition to resolve the gauge hierarchy or naturalness problem.

In this work, we have shown that it is possible to localize the SM fermions in the bulk using the higher curvature dilaton coupled gravity set-up without invoking any external scalar field in the bulk. This naturally explains the origin of localization of left handed fermions on the visible brane whereas the right handed fermionic modes get delocalized and obtain their peak inside the bulk. We have also obtained the effective 4D mass term on the visible brane which depends on the GB coupling parameters and dilaton coupling. Different values of these parameters yield different fermion mass values leading to a possible explanation of flavour hierarchy among standard model fermions.Our work thus offers a natural origin of the standard model flavour hierarchy in a warped braneworld model through the higher order quantum corrections over Einstein's gravity.

Acknowledgments: SC thanks Department of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai for providing Visiting Research Fellowship. SC and JM thanks Indian Association for the Cultivation of Science (IACS) for various support during the work.

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [hep-ph/9905221]; L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690 [hep-th/9906064].
[2] G. Weiglein *et al.* [LHC/LC Study Group Collaboration], Phys. Rept. **426** (2006) 47 [hep-ph/0410364].
[3] J. P. Skittrall, Eur. Phys. J. C **60** (2009) 291 [arXiv:0809.4383 [hep-ph]].
[4] E. De Pree and M. Sher, Phys. Rev. D **73** (2006) 095006 [hep-ph/0603105].
[5] G. Aad *et al.* [ATLAS Collaboration], New J. Phys. **15** (2013) 043007 [arXiv:1210.8389 [hep-ex]].
[6] Y. Tang, JHEP **1208** (2012) 078 [arXiv:1206.6949 [hep-ph]].
[7] A. Das and S. SenGupta, arXiv:1303.2512 [hep-ph].
[8] B. Bajc and G. Gabadadze, Phys. Lett. B **474** (2000) 282 [hep-th/9912232].
[9] J. L. Lehners, P. Smyth and K. S. Stelle, Nucl. Phys. B **790** (2008) 89 [arXiv:0704.3343 [hep-th]].
[10] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, Phys. Rev. D **62** (2000) 084025 [hep-ph/9912498].
[11] R. Koley and S. Kar, Class. Quant. Grav. **22** (2005) 753 [hep-th/0407158].
[12] N. Arkani-Hamed, Y. Grossman and M. Schmaltz, Phys. Rev. D **61** (2000) 115004 [hep-ph/9909411].
[13] D. E. Kaplan and T. M. P. Tait, JHEP **0111** (2001) 051 [hep-ph/0110126]. S. Choudhury, J. Mitra and S. SenGupta, JHEP **1408** (2014) 004 [arXiv:1405.6826 [hep-th]].
[14] Y. Grossman and M. Neubert, Phys. Lett. B **474** (2000) 361 [hep-ph/9912408].
[15] S. Choudhury and S. Sengupta, JHEP **1302** (2013) 136 [arXiv:1301.0918 [hep-th]]; S. Choudhury, S. Sadhukhan and S. SenGupta, arXiv:1308.1477 [hep-ph]; S. Choudhury and S. SenGupta, Eur. Phys. J. C **74** (2014) 11, 3159 [arXiv:1311.0730 [hep-ph]].
[16] P. Dey, B. Mukhopadhyaya and S. SenGupta, Phys. Rev. D **81** (2010) 036011 [arXiv:0911.3761 [hep-ph]].
[17] S. J. Huber and Q. Shafi, Phys. Rev. D **63** (2001) 045010 [hep-ph/0005286]; S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, Phys. Rev. D **62** (2000) 084025 [hep-ph/9912498]; C. H. V. Chang and J. N. Ng, Phys. Lett. B **488** (2000) 390 [hep-ph/0006164]; H. Davoudiasl, B. Lillie and T. G. Rizzo, JHEP **0608** (2006) 042 [hep-ph/0508279].